National intangible capital augmented production function

OR

SHARPENING THE PRODUCTION FUNCTION EDGE

Motto

It takes something more than the usual
"willing suspension of disbelief"
to talk seriously of the aggregate production function.

Robert Solow
Technical Change and
the Aggregate Production Function
1957

The simple C-D production function

The basic production function in focus is the well known Cobb-Douglas production function establishing the relationship between production, capital, labor and total factor productivity, generally available technology, as:

$$Y = A K^a L^b$$

Where

Y = Production per year ($ or $h)

K = Capital hours per year ($h)

L = Labor hours per year (lh)

A = Total factor productivity ($/h^{ab} or $^{1-ab}$) and production share elasticites a and b with a + b = 1

Production share elasticities

Production share elasticities a and b are calculated via output Y, capital employed K and cost of labor input Lw (w = cost of labor per unit):

For capital

a' = K/Y ; K = capital employed, e.g. fixed capital

For labor

b' = Lw/Y ; Lw = cost of labor input

Finally

a = a'/ (a' + b') ; a = capital relative share of input

b = b' / (a' + b') ; b = labor relative share of input

Which gives

a + b = 1 ; Condition guarantee CRS

CRS = Constant Return to Scale

Simplicity

The simplicity lies in the fact that the C-D production function presupposes:

- Uniform production shares (a and b) for capital and labor, e.g. the exponents a and b covers the whole capital and all workforce unchanged.
- Capital and labor production inputs (K and L) are non-qualitative, e.g. the quality of capital K and labor L aspect is omitted.

Thus, when applying the C-D production function on a macro economic, national, level we need both to aggregate and augment:

- We aggregate by recognizing production share differences, i.e. by recognizing differences in urban and rural areas etc..
- We augment by recognizing quality variations, i.e. national and regional variations in human capital affecting productivity of labor and capital etc..

The aggregating point of view is well known and elaborated, most notably by Robert Solow (e.g. starting 1957), but the augmenting approach is yet very much vague.

Aggregate

Aggregation is here performed using a (3x3) 4D matrix:

- D1 = urban – suburban – rural % of population POP
- D2 = service – industry – agriculture % of workforce L
- D3 = urban – suburban – rural % of GDP formation
- D4 = service – industry – agriculture % of GDP formation

Using this (rather simple) approach capital K and labor L production shares (elasticities a and b) will be different for urban, suburban and rural areas and also within service, industry and agriculture. E.g. we aggregate output Y via 3x3 = 9 different national economic structures.

Simplicity is here chosen – and argued– as general cross country comparison and evaluation based on broad general national statistical data is most wanted. Simplicity is also chosen to bring out (conceptual) knowledge economy structural significance.

Augment

Augmenting is performed using a 3D matrix:

- D1 = urban – suburban – rural % of population POP
- D2 = service – industry – agriculture % of workforce L
- D3 = FC-HC-MC-PC-RC FC and NIC index values

with FC and national intangible capital NIC as:

FC	Financial capital	HC	Human capital
PC	Process capital	MC	Market capital
RC	Renewal and innovation capital		

Using this approach capital K and labor L will be augmented (enhanced or hampered) by national financial and intangible capital, FC and NIC, quality measures on each aggregate level. E.g. we augment output Y via 5 main quality measures applied on capital and labor within 9 different economic structures.

The aggregate augmented production function

Together
the aggregate augmented production function embrace

*9*5 = 45*

distinct national "knowledge economy" structures.

NIC indexes

Modified and enhanced NIC indexes are based on Edvinsson and Lin (2010).

NIC basic (lower) indexes are here modified using following principles:

- Where % of GDP (PPP) is the nominal (original) value this is adjusted by GDP % growth. This corrects the situation i.e. where investments in education as % of GDP is 3 % for two years but GDP is declining / growing.
- Where per capita is the nominal value this is (in most cases) changed to per GDP. This adjusts i.e. researchers per 1000 FTE (full time equivalent) or patents in force per million inhabitants for countries with extremely large / small populations and stresses the relation to GDP formation, not population size.
- Where 15 to 65 years is the criteria this is (in most cases) adjusted by effective entry into labor age to effective retirement age. This fine-tunes i.e. dependency ratios.

In addition:

- Where nominal values are survey data at least three (3) nominal values (surveys) are combined to form one basic index. This, to some extent, corrects the subjectivity question involved. I.e. when skilled labor is aggregated (as i.e. a weighted average) with foreign high-skilled labor and international expertise the aggregated index possesses higher (subjective) reliability.

NIC HC and MC indexes

Human Capital index HC

1. Skilled labor*
2. Employee training*
3. Secondary education up enrollment
4. Pupil-teacher ratio
5. Public expenditure on education
6. 15-64 years old population
7. Qualified engineers*
8. Students PISA performance
9. Human Development Index
10. Gender equality
11. Years of education
12. R&D researchers

Market Capital index MC

1. Corporate tax encouragement*
2. Cross border venture*
3. Openness of culture*
4. Transparency of government policies*
5. Image of your country*
6. Capital availability*
7. Trade to GDP ratio (exports + imports)
8. Current account balance %GDP
9. Investment flows %GDP
10. Country credit rating
11. Investment risk
12. Globalization index

NIC PC and RC indexes

Process Capital index PC

1. Business competition environment*
2. Government efficiency*
3. Computer per capita + Mobile subscribers
4. Internet subscribers + Broadband subscribers
5. Convenience of establishing new firms + start up days*
6. Goods & services distribution efficiency*
7. Overall productivity
8. Unemployment % + Youth unemployment %
9. Consumer price inflation
10. Health & environment
11. Corruption
12. Freedom of speech

Renewal Capital index RC

1. Business R&D spending
2. Basic research*
3. R&D spending/GDP
4. R&D US$ per capita
5. IP right protection*
6. Utility Patents/ R&D expenditure
7. Cooperation between corporations and university*
8. Scientific articles
9. Patents per capita (USTPO+EPO)
10. Entrepreneurship*
11. Development & application of technology*
12. Venture capital*

Financial capital FC

Financial capital index FC is put into action and calculated using following basic indexes:

- 1. GDP per capita at PPP
- 2. Government surplus/deficit % of GDP
- 3. Real government debt % growth rate
- 4. Foreign Debt % of GDP (government and business)
- 5. US$ currency exchange rate (or weighted US$ & Euro)
- 6. Long term interest rate
- 7. Effective country long term debt interest rate
- 8. Gross fixed capital formation per capita
- 9. Stock market capital formation as % of GDP
- 10. FDI inwards % of GDP
- 11. FDI outwards % of GDP
- 12. Value of special natural and financial resources /services (export - import as % of GDP)

(3, 4, 12) are of special importance as they explain anomalous GDP formation, i.e. Norway benefiting hugely through its oil resources and Greece outwardly from its growing debts (2001 – 2009) etc..

Estimating main variables Y, K, L, a and b

Main factors affecting the Cobb-Douglas production function are:

- 1. GDP at PPP (nominal)
- 2. Labor force L (nominal)
- 3. Gross fixed capital formation K (nominal)
- 4. Working hours per year (nominal)
- 5. Total wages / compensations (nominal)
- 6. Compensation per hour w (nominal)
- 7. National debt (nominal)
- 8. FDI inwards (nominal)
- 9. FDI outwards (nominal)
- 10.1-3. Sectoral breakdown % of GDP (part of EMS estimates)
- 11.1-3. Population demographic structure (Urban-Rural)
- 12. Energy consumption TOE (part of EMS estimates)
- 13. Energy intensity as MTOE/GDP (part of EMS estimates)
- 14. Key Natural Resources (nominal)
- 15. Key Financial services (nominal, part of EMS estimates)

Note: EMS stands for Energy, Materials and Services.
TOE stands for Tons of Oil Equivalent and MTOE Millions TOE.

Normalizing NIC indexes

Normalized NIC indexes, basic and composite, possess the following characteristics:

- Nominal values are normalized using most appropriate normalizing method
  - Likert like (survey) data sets are normalized using $x' = (x-AVG)/s$
    - To enforce STDEV spread power enhancing may first be applied
  - Nominal data sets with extremes (extreme positive and/or negative values) are normalized using Sigmoid (like) $x' = x/SQRT(x^2 + a)$ where $a = AVG^2$ or
  - Applying Sigmoid $x' = 1/(1 + e^{-a(x-MEDIAN)})$ where $a = 4/(Q3-MEDIAN)$
- STDEV spread is then adjusted to meet
  - Min/RANGE (= MAX – min) equals original data and
  - Median value = 5

Practically this produces index values with 0 < index value < 10 around a constant median value = 5. However, index values allow and show growth and has no principal upper limit, 0 being the principal lower limit.

Note: Only country data (basic indexes) covering the full time period (here 2001 – 2015) is used. This is done to avoid irregular changes when (if) clearly above / under country average data is added and aggregated in the midst of the time period. If such data is added this leads to unexplainable behavior of the aggregated index. I.e. the index gets notably higher or lower in the midst of the period without any real reason. Interpolation is however used, when appropriate, to replace (single or two) missing data values.

The residual approach

A residual is generally a quantity left over at the end of a process.

In a production function (e.g. Cobb-Douglas production $Y = A K^a L^b$) the residual A (><1) is what can not be explained or deduced via components in the process (e.g. “more than”, excess, production compared to $Y' = K^a L^b$ (or less)).

In this work we perceive the residual as a realistic and plausible gate way to accurately estimate (the effect of) intangibles (and the environment) involved in production and productivity as it incorporates, what K and L just can’t explain.

We calculate the residual as:

$$A = TFP = Y / (K^a L^b)$$

Note: Aggregation denotations omitted for sake of clarity.

A

Instead of capitalizing K
by adding new capital (e.g. Corrado 2005)
or enhancing labor L (e.g. Romer 1992)

the aggregate augmenting
focus on cracking TFP itself

A

SIC!

Decomposing the residual

We allow for a general (FP_{ext}) and country specific (FP_{int}) part of factor productivity FP (comp. Lev 2003) together with FC_{int} as

$$TFP = A = (FP_{ext})^e (FC_{int})^f (FP_{int})^g \quad ; e + f + g = 1$$

Production shares for (FP_{ext}), (FC_{int}) and (FP_{int}) are e, f and g respectively with e + f + g = 1. Using national intangible capital NIC index values and denoting

$$TFP_{all} = A_{all} = Average(A_i) \quad ; i = 1, \ldots countries$$
$$FC_{all} = F_{all} = Average(FC_i) \quad ; i = 1, \ldots countries$$
$$NIC_{all} = N_{all} = Average(NIC_i) \quad ; i = 1, \ldots countries$$

together with MTFP representing effects via markets and total factor productivity (e.g. what we don't grasp we call general environmental effects) we estimate

$$A_{all} = (MTFP)^e [(A_{all} / F_{all}) FC_i]^f [(A_{all} / N_{all}) NIC_i]^g$$

where country specific (external) FP_{ext} = MTFP, country specific (internal) FC and country specific (internal) FP_{int} = NIC.

Note: Aggregation denotations omitted for sake of clarity.

Decomposing NIC

Productivity shares for HC, MC, PC and RC are k, l, m and n with k + l + m + n = 1, e.g.

$$g = (k + l + m + n)\, g$$

and the estimate

$$A_i = (MTFP_i)^e [(A_i / F_{all}) FC_i]^f [(A_i / N_i) HC_i^k \ MC_i^l \ PC_i^m \ RC_i^n]^g$$

with country specific (external) $MTFP_i$, country specific (internal) FC_i and (internal) $NIC_i = N_i$.

Note: Aggregation denotations omitted for sake of clarity.

Production shares

The production shares c(i) for various variables in the aggregate NIC augmented Cobb-Douglas production function are:

c1 MTFP $(Y\ MTFP_i)^e$

c2 FC $[Y\ FC_i]^f$

c0 NIC $[Y\ (A_{all} / N_{all})\ NIC_i]^g$ or

$[Y\ (A_i / N_i)\ HC_i^k\ MC_i^l\ PC_i^m\ RC_i^n]^g$

c3 HC $[Y\ (A_i / N_i)^k\ HC_i^k]^g$

c4 MC $[Y\ (A_i / N_i)^l\ MC_i^l]^g$

c5 PC $[Y\ (A_i / N_i)^m\ PC_i^m]^g$

c6 RC $[Y\ (A_i / N_i)^n\ RC_i^n]^g$

Note: Aggregation denotations omitted for sake of clarity.

Contributions to GDP formation

Using the calculated production shares we get the contributions to GDP formation to be:

With SC = c0 + c1 + c2

MTFP	Y c1/SC
FC	Y c2/SC
NIC	Y c0/SC

With SC = c1 + … + c6

NIC	Y s0/SC or
	Y (s3 + s4 + s5 +s6)/SC
HC	Y c3/SC
MC	Y c4/SC
PC	Y c5/SC
RC	Y c6/SC

Decomposing NIC composite indexes

Using interim results for composite, aggregated indexes, i.e. using HC_i^k , and denoting related HC basic indexes HC1-n production shares for the primary n basic indexes can be calculated through:

$$HC_i^k = [\ HC1^r \ldots HCn^s\ ]^k \quad ; r + \ldots + s = 1$$

Or

$$HC_i = [\ HC1^r \ldots HCn^s\ ]$$

Then following the same procedure as before, when calculating NIC and NIC composite indexes production shares and contribution to GDP formation the production shares and contributions to GDP formation for basic indexes can be estimated.

The approach is applicable for FC, HC, MC, PC and RC alike.

NIC as driver of GDP

Decomposing NIC composite indexes opens up the gate way
to pin point and assess cost efficient NIC drivers of GDP
as both the monetary value of the driver and its impact in GDP formation can

(in most cases)

be estimated.

Practical note

As the material at hand

61 countries

15 years

is rather limited and as it is of essence to estimate especially

production share elasticities and

related production share elasticities for NIC etc.

on a

year to year and

country to country

specific accuracy level

regression analysis is not very suitable!

As an alternative to find the desired precision genetic optimizing is used. This can be considered a credible choice as genetic optimizing is designed to handle discreet events with only slight differences in similar patterns within a limited population.

To put it an other way:

It takes something more than the usual
"willing suspension of disbelief"
to talk seriously of the aggregate augmented production function.